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THE DISTRIBUTION OF AEROSOL PARTICLES DOWNSTREAM IN A TURBULENT FLUID JET (U)

by

Kathy I. Leary* and Stanley B. Mellsen

PCN No. 13E10

December 1982

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* Chemistry Section Research Assistant, Summer 1982



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ABSTRACT

→ A mathematical model has been developed to predict the downstream distribution of particles in a fluid jet. The particles were initially assumed to be evenly dispersed over a cross-section of the circular jet, near the source of the jet, and, with computer-aided methods, the co-ordinates of the particles at a specified distance downstream were determined. The computer program was also built in such a manner that the paths of specific particles, from their initial upstream position to their final position downstream, could be completely followed. Some graphical results of these paths are presented for various particle sizes. ←

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NOTATION

$b_{1/2}$	half the width of a circular jet at half depth, cm
d	particle diameter, cm
D	jet diameter where particles are initially positioned in flow stream, cm
k	kinematic momentum of circular jet, $k = 2\pi \int_0^{\infty} u^2 y dy$
L	jet radius where particles are initially positioned in flow stream, cm
t	time, seconds
u	local fluid velocity, cm s^{-1}
u_x	axial component of fluid velocity, cm s^{-1}
u_r	radial component of fluid velocity, cm s^{-1}
U	center-line velocity of fluid at any distance downstream, cm s^{-1}
U_0	center-line fluid velocity at x_0 , cm s^{-1}
x_0	axial distance from point source of jet where particles are initially positioned in flow stream, cm
x	co-ordinate of particle position parallel to center-line flow direction; axial co-ordinate, cm
y	co-ordinate of particle position perpendicular to center-line flow direction; radial co-ordinate, cm
σ	particle density, g cm^{-3}
μ	absolute viscosity of fluid, poise
ρ	fluid density, g cm^{-3}
ϵ_0	virtual kinematic viscosity of turbulent flow, poise

The following are dimensionless:

C_D	drag coefficient for spheres
K	particle inertia parameter
Re	spherical particle slip Reynolds number
Re_0	spherical particle Reynolds number in free stream

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NOTATION (Cont'd)

T	time tU/L
\bar{u}_x	axial component of fluid velocity u_x/U_0
\bar{u}_y	radial component of fluid velocity u_y/U_0
\bar{v}_x	$d\bar{x}/dT$, parallel component of particle velocity
\bar{v}_y	$d\bar{y}/dT$, transverse component of particle velocity
η	parameter used in calculation of fluid velocity
ϕ	parameter formed by combining Re_0 and K

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APPENDIX

Listing of the Computer Program with a Sample Solution.

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INTRODUCTION

Previous studies have been made (Mellsen, 1978 and 1979) involving streams of suspended particulates in such a way that impaction forces of airborne particles and collection efficiencies of specific samplers could be determined. The flow considered in the present study is of this same type; that is, it involves a fluid flow with suspended particles moving along with the flow. As a result, the same mathematical equations as were previously used may be used here to describe the motion of the particles in the fluid. In doing so, the velocity, as well as the position of each particle at any distance downstream can be determined.

The specific flow which is to be studied here is that of a turbulent circular jet with particles suspended across a cross-section of the jet. By developing a mathematical model, the radial and axial co-ordinates of the particles at a specific downstream distance will be determined, thus the actual particle distribution will be known. Also, the model is to be devised in such a fashion that the paths of particular particles may be completely followed from their initial upstream position to their final downstream position.

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The information as revealed by this model will prove to be of some value from the chemical defence point of view in that it is of interest to know how particulate agents, i.e., irritants, such as CS and CR disperse in a flow field such as that described by a circular jet. The solution may also be applied to situations involving fluid-particle mixtures flowing from smoke stacks and/or exhaust pipes.

DEFINITION OF THE PROBLEM

When a jet leaves a small circular opening it causes surrounding fluid to be drawn in and mixed with the jet itself (Fig. 1). This results in an increase of volume of flow with distance from the orifice and a downstream cross-section of the flow field which is much larger than a cross-section nearer the source of the jet.

The problem to be considered here involves putting particles into a circular jet flow somewhere upstream and determining their positions downstream. We would like this situation to simulate a fluid-particle flow from a tube, with particles initially distributed across the mouth of the tube, however the mathematics describing the flow field assumes the jet as coming from a point source. In order to account for this, the particles are put in the flow, at the velocity of the flow, a short distance downstream from this issuing point. Their paths are then followed from this initial point onwards.

The distance from the source where the particles are initially distributed uniformly across the cross-section is defined by how much of the jet velocity profile is to be studied. It has been found that a distance, x_0 of $5.896D$ from the point source encompasses the width of the jet at half depth (Schlichting, 1968). That is, for this x_0 value the velocity of the particles which are located on the circumference of the circular cross-section will be one-half that of the center-line velocity (Fig. 2).

By increasing x_0 , a smaller amount of the jet angle is covered and the velocity of the particles on the circumference will be more than one-half of the center-line velocity. When the value of x_0 is less than that of $5.896D$ the opposite situation arises; a larger amount of the jet angle is covered and particle velocity on the circumference becomes smaller.

Thus, the basic problem is to choose an appropriate x_0 according to the portion of the jet angle to be studied, and thereby establish the starting point of the particle paths. Having done this, the paths are to be traced to a specified distance downstream where the co-ordinates of each particle are to be determined.

PARTICLE MOTION

The motion of an individual spherical particle has been found (Batchelor, 1967) to be defined by the following ordinary differential equations:

$$\frac{d\bar{v}_y}{dT} = \frac{C_D Re(\bar{u}_y - \bar{v}_y)}{24K} \quad (\text{Eq. 1})$$

$$\frac{d\bar{v}_x}{dT} = \frac{C_D Re(\bar{u}_x - \bar{v}_x)}{24K} \quad (\text{Eq. 2})$$

$$\text{where } Re = Re_0[(\bar{u}_y - \bar{v}_y)^2 + (\bar{u}_x - \bar{v}_x)^2]^{1/2} \quad (\text{Eq. 3})$$

$$K = \frac{\rho d^2 U_0}{18\mu L} \quad \begin{array}{l} \text{particle inertia parameter} \\ \text{(Stokes number)} \end{array} \quad (\text{Eq. 4})$$

$$Re_0 = \frac{U_0 d \rho}{\mu} \quad \begin{array}{l} \text{free stream Reynolds number} \end{array} \quad (\text{Eq. 5})$$

The variables are defined on the notation pages at the beginning of this report and the basic geometry of the flow system is illustrated in Figure 1.

Both Equations 1 and 2 involve the term $C_D Re$. This presents a slight problem because C_D , the drag coefficient of the sphere, has been found to be a function of the Reynolds number Re . For example,

$$C_D = \frac{24}{Re}$$

for a situation involving Stokes flow. However, since both C_D and Re contain a velocity term it is more convenient to use

$$Re = \frac{C_D Re^2}{24}$$

when considering Stokes flow. This is because the velocity squared term contained in the denominator of the definition of C_D is cancelled by the velocity squared term in Re^2 and $C_D Re$ may then be found by dividing $C_D Re^2$ by Re .

As a result, the general expression becomes $Re = f(C_D Re^2)$ and in actual fact the drag coefficient is available in the form of explicit empirical equations. These equations, are (Davies, 1945):

$$Re = - \frac{C_D Re^2}{24} - 2.3363 \times 10^{-4} (C_D Re^2)^2 + 2.0154 \times 10^{-4} (C_D Re^2)^3 - 6.9105 \times 10^{-9} (C_D Re^2)^4 \quad (\text{Eq. 6})$$

for $Re < 4$ or $C_D Re^2 < 140$

$$\log_{10} Re = -1.29536 + 9.86 \times 10^{-1} (\log_{10} C_D Re^2) - 4.6677 \times 10^{-2} (\log_{10} C_D Re^2)^2 + 1.1235 \times 10^{-3} (\log_{10} C_D Re^2)^3 \quad (\text{Eq. 7})$$

for $3 < Re < 10^4$, or $100 < C_D Re^2 < 4.5 \times 10^7$

The assumptions that must be made when using any of the equations which were introduced in this section include —

- (a) negligible effects of gravity and/or electrostatic forces,
- (b) monodisperse spherical particles with diameter very small as compared to the diameter of the initial upstream cross-sectional area, and
- (c) free stream flow that is steady, incompressible and irrotational.

THE TURBULENT CIRCULAR JET

The velocity components of a turbulent circular jet have been defined (Schlichting, 1968) as:

$$\left. \begin{aligned} u_x &= \frac{3}{8\pi} \frac{k}{\epsilon_0 x} \frac{1}{(1 + \frac{1}{4}\eta^2)^2} \\ u_y &= \frac{1}{4} \sqrt{\frac{3}{\pi}} \frac{\sqrt{k}}{x} \frac{\eta - \frac{1}{4}\eta^2}{(1 + \frac{1}{4}\eta^2)^2} \end{aligned} \right\} \quad (\text{Eq. 8})$$

where $\eta = \frac{1}{4} \sqrt{\frac{3}{\pi}} \frac{\sqrt{k}}{\epsilon_0} \frac{y}{x}$

The curve which results from these equations has been found to agree quite favorably with the first theoretical consideration of a circular jet as done by W. Tollmien and the experimental results of H. Reichardt (Figure 24.8, Schlichting).

In applying Equations 8 to the problem considered here it is necessary to find numerical values for particular constants such as the empirical constant, \sqrt{k}/ϵ_0 , and/or the constant kinematic momentum, k . In order to do this, the theory as presented (Schlichting) must be employed to modify the equations of u_x and u_y to more suitable forms. These modifications are now to be introduced.

Due to measurements obtained by H. Reichardt, half the jet width at half the depth of the velocity distribution at any point x is given by $b_{1/2} = 0.0848x$. This is actually half the width of the jet at the point where, for each distribution, the minimum velocity in the x direction is one-half of the maximum velocity which occurs.

According to Schlichting, the point at which the fluid velocity is one-half the maximum fluid velocity, $\eta = 1.286$ and $b_{1/2} = 5.27x \epsilon_0/\sqrt{k}$. By equating these two expressions of $b_{1/2}$, the result is

$$\frac{\epsilon_0}{\sqrt{k}} = 0.0161$$

so that \sqrt{k}/ϵ_0 , which occurs in u_x and η , is $1/0.0161$.

The equations we now have are:

$$u_x = \frac{3}{8\pi x} \left(\frac{1}{0.0161} \right) \frac{1}{(1 + \frac{1}{4}\eta^2)^2} \sqrt{k}$$

$$u_y = \frac{1}{4} \sqrt{\frac{3}{\pi}} \frac{1}{x} \frac{\eta - \frac{1}{4}\eta^2}{(1 + \frac{1}{4}\eta^2)^2} \sqrt{k}$$

$$\eta = \frac{1}{4} \sqrt{\frac{3}{\pi}} \left(\frac{1}{0.0161} \right) \frac{y}{x}$$

It can be seen that for any co-ordinate consisting of an x and y value, η can be found and the only unknown quantity left to be determined is \sqrt{k} found in u_x and u_y .

It has been derived from substitution that

$$\sqrt{k} = 1.59 b_{1/2} U$$

or
$$\sqrt{k} = 1.59(0.0848x)U = 0.1348xU$$

where U is the center-line velocity at any distance x . Not knowing this value, it appears that it would be more convenient to express \sqrt{k} in terms of U_0 — the velocity of the fluid at x_0 , where x_0 is the distance downstream from the issuing point of the jet that is considered the starting point of the solid particles.

It is known that for a free turbulent circular jet the center-line velocity U is proportional to x^{-1} therefore

$$U = \frac{c}{x}$$

and
$$U_0 = \frac{c}{x_0} \quad \text{where } c \text{ is an arbitrary constant.}$$

By obtaining an expression for x_0 , then, knowing U_0 , the constant can be found as can the appropriate value of U for any x .

Assuming the situation as illustrated in Figure 2, then $b_{1/2} = 0.5D$. Also, $b_{1/2} = 0.0848x_0$ as was previously introduced. Thus

$$x_0 = \frac{0.5D}{0.0848} = 5.896D \quad (\text{Eq. 9})$$

and
$$c = 5.896DU_0$$

The velocity U at any downstream distance x may now be expressed in terms of U_0 , D and x :

$$U = \frac{5.896DU_0}{x}$$

and the simplified expression for \sqrt{k} becomes

$$\sqrt{k} = 0.795DU_0$$

By substituting this into u_r and u_z , the resulting fluid velocity components are:

$$u_r = \frac{3}{8\pi x} \left(\frac{1}{0.0161} \right) \frac{1}{(1 + \frac{1}{4}\eta^2)^2} (0.795DU_0)$$

$$u_y = \frac{1}{4} \sqrt{\frac{3}{\pi}} \frac{1}{x} \frac{\eta - \frac{1}{4}\eta^2}{(1 + \frac{1}{4}\eta^2)^2} (0.795 D U_0)$$

Thus, for any constants D and U_0 (which are analogous to OD and UF of the computer program used to solve the problem), and for any x and y co-ordinate, η may be calculated as well as can u_x and u_y .

Each fluid velocity component was normalized in the solution to the center-line fluid velocity which prevailed at x_0 . This was done so that the initial velocity in the x direction of the fluid at the center, and thus the center particle, was always 1.0. All other velocities could easily be compared to this value.

Finally, as an overall conclusion to these previous substitutions and derivations, the normalized equations which were actually used to find the velocity components of a turbulent circular jet for the situation of this problem are:

$$\left. \begin{aligned} \bar{u}_x &= \frac{u_x}{U_0} = \frac{3}{8\pi x} \left(\frac{1}{0.0161} \right) \frac{1}{(1 + \frac{1}{4}\eta^2)^2} (0.795 D) \\ \bar{u}_y &= \frac{u_y}{U_0} = \frac{1}{4} \sqrt{\frac{3}{\pi}} \frac{1}{x} \frac{\eta - \frac{1}{4}\eta^2}{(1 + \frac{1}{4}\eta^2)^2} (0.795 D) \end{aligned} \right\} \text{(Eq. 10)}$$

where, as before

$$\eta = \frac{1}{4} \sqrt{\frac{3}{\pi}} \left(\frac{1}{0.0161} \right) \frac{y}{x}$$

COMPUTER METHODS OF SOLUTION

The computer program which was built to solve the problem considered here consists of various subprograms incorporated into the main program itself. Each of these subprograms are briefly described in the program listing and the whole program is documented in such a way that the step-by-step procedure may be quite easily followed. Refer to the Appendix for a listing of the program and a sample solution.

A total of fourteen constants are to be supplied by the user and these are read in as input data. Each of these variables, as well as most of the others used throughout the

program are described at the beginning of the listing so that they may be easily referred to while running the program.

In order to establish the initial upstream starting point, a constant AF is to be read in and used in the statement $XZ = 5.896 \cdot OD / RADZ \cdot AF$. The value of AF provided by the user depends upon the angle of the jet velocity profile to be studied. With $AF = 1.0$ the jet encompasses the jet width at half the depth of the profile. See Figure 2 for an illustration of the situation resulting when $AF = 1.0$ and refer to the previous section entitled "Definition of the Problem" for a more complete description of this value of x_0 . When AF is less than 1.0, then a different value of x_0 results and a wider portion of the velocity profile is covered; when AF is greater than 1.0, a smaller jet angle than that at half depth is covered. Thus by varying the input value of AF different portions of the jet velocity profile may be studied according to the user's preference.

One of the most important subroutines which was built into the solution program is that of SUBROUTINE SBM22. As is described in the listing, this uses the fourth-order Runge Kutta method to integrate a specified number of ordinary differential equations. In this case, the ordinary differential equations which are to be solved by SUBROUTINE SBM22 are

$$\frac{d\bar{v}_x}{dT}, \frac{d\bar{v}_y}{dT}, \frac{d\bar{x}}{dT}, \frac{d\bar{y}}{dT}$$

which are analogous to DG(1), DG(2), DG(3) and DG(4) respectively of the computer program. By integrating these equations, the velocity and position of the particle is obtained (note that DG(1) and DG(2) represent Equations 1 and 2).

The values of Re and K of Equations 1 and 2 are easily found for each new step by direct substitution of previously determined values into Equations 3, 4 and 5. However, the value of CDRE is found by the subprogram FUNCTION CDRE. This is done by calculating, for each step, the numerical solution of the definitive empirical equations (Eqs. 6 and 7). By using Newton's method for finding the zero of a function, these numerical solutions are determined and the value of CDRE can be obtained.

This subprogram, FUNCTION CDRE, presents the only visible error of the whole program. The procedure used involves an iterative process which is terminated only when a certain condition is met, namely, that

$$\left| \frac{x_{k+1} - x_k}{x_{k+1}} \right| \leq \epsilon$$

where ϵ is any small number. When this condition is not met then a message of NO CONVERGENCE is printed by the computer and the value of CDRE is taken to be what was previously stored there. Although the number of iterations (100) is quite high, there are some situations where the initial estimate doesn't converge and the message is printed out. In these cases, the true value of CDRE is, in fact, not what it is taken to be and a small amount of error is introduced into the calculations by assuming an incorrect value. However, since the actual computed values of CDRE change very little from one time increment to the next, then this error is assumed to be negligible and the message may be, for practical purposes, ignored.

The program was written for a large number of diverse situations however one limitation which was discovered is that the smallest particle diameter which could be input is 20 μ . At a diameter of 10 μ , an overflow problem in the computer calculations resulted so no solution could be obtained. Other limitations of the program have not yet been discovered.

As a small addition to the solution, the average number of particles per unit area (cm^2) at the initial cross-section is computed, as it also is at the cross-section downstream. It may be noted that the upstream concentration is in fact the actual number of particles per square centimeter since they (the particles) are uniformly distributed over the cross-section. However, upon reaching the downstream distance, the particles will no longer be uniformly dispersed and the resulting concentration represents an average of particles per square centimeter.

RESULTS AND DISCUSSION

A sample solution for one set of input data is shown after the program listing in the Appendix. Many more calculations were made using this program to produce the graphical results shown in Figures 3 to 7.

The plots of Figures 3, 4 and 5 are those of the actual particle paths for particle diameters of 100 μ , 50 μ and 20 μ . It can be seen that the downstream distribution does not vary much with particle size, however, the larger particulates tended to become distributed a bit further away from the center of the flow. Even though the difference is small it indicates that the larger particles were less inclined to be influenced by the surrounding fluid which is drawn in as the jet spans out downstream. If this is the case, the momentum of these larger particles would cause them to flow out and away from the

center more than the smaller particles and as a result, they would have greater y co-ordinates. It must be noted that these graphs were plotted for a value of AF equal to 0.3217 which was chosen so that most of the velocity profile was covered (this was determined for v_r of the outside particle equal to 0.0 at the initial position).

There are no experimental results available to compare with the mathematical results presented in Figures 3, 4 and 5. Since the model contains many assumptions, including steady, irrotational flow and the starting location of the particles, the results should be tested by experiment to establish their validity. Some errors, however, could possibly have been introduced into the solution by computer round-off but these are assumed to be negligible.

The graphs of Figures 6 and 7 are families of curves showing the radial co-ordinate of the outside particle as a function of the inertia parameter K . (The outside particles are those which are located on the circumference of any circular cross-sectional area of the jet which engulfs all particles.) These curves are plotted for a series of values of ϕ , a dimensionless parameter independent of particle size and equal to Re_0^2/K .

Figure 6 shows the results for $AF = 1.0$ which means that all of the particles are assumed to be distributed across the width of the jet at half the depth of the velocity profile. At this point, as was previously mentioned, the initial velocities of the outside particles (in the x direction) are one-half the velocity of the center particle. Figure 7 shows the results for $AF = 0.5$ which covers a wider portion of the jet angle than when $AF = 1.0$. At this angle, the velocities of the outside particles are found to be about 0.14 times that of the center particle thus confirming that a wider portion of the profile is indeed covered.

In these two figures each curve was fitted by eye through several data points obtained by applying the computer program to various particle diameters. As a result, by knowing the particles' free stream Reynolds number Re_0 and the inertia parameter K then by studying the appropriate curve, the radial co-ordinates of the outside particles at a specific distance downstream may be estimated. Also, for any value of ϕ which falls between those plotted in the figures, the method of interpolation provides a means of approximating the downstream radial co-ordinate.

As a last point to note, the results as given in the Appendix indicate that although both the particles and the fluid have the same initial velocity, the velocity of the fluid tends to decrease faster than the particles as they are observed downstream. This can be explained by considering the effect of the fluid on the particles in the following way:

In the presence of a fluid which is continually decreasing in velocity, as is the case here, particles moving in the same direction as the fluid will have a drag force acted upon them. Although the particles will begin to slow down very quickly they will still have a greater velocity than the fluid because of the inertia effects which they experience. As the distance downstream is increased however, the difference between particle velocity and fluid velocity is narrowed (refer to columns of VX' and UX' of the solution given in the Appendix). This is due to the fact that the fluid velocity is changing more slowly as the downstream distance is increased.

CONCLUSIONS

In some situations it may be of interest to know how particulates disperse in a fluid-particle circular jet. As an attempt to find some answers a mathematical model has been developed which predicts the distribution of particles, by specifying their radial and axial co-ordinates, at any distance downstream. This model also enables the paths of specific particles to be completely followed from their initial upstream position to their downstream position.

A computer program was written to provide quick and easy solutions to this mathematical model. Also, the computer solution was devised in such a manner that a wide variety of flow situations can be considered; this depends only on the data which is provided as input by the user. However, the output of this program should be validated by experimental data since the model contains many assumptions.

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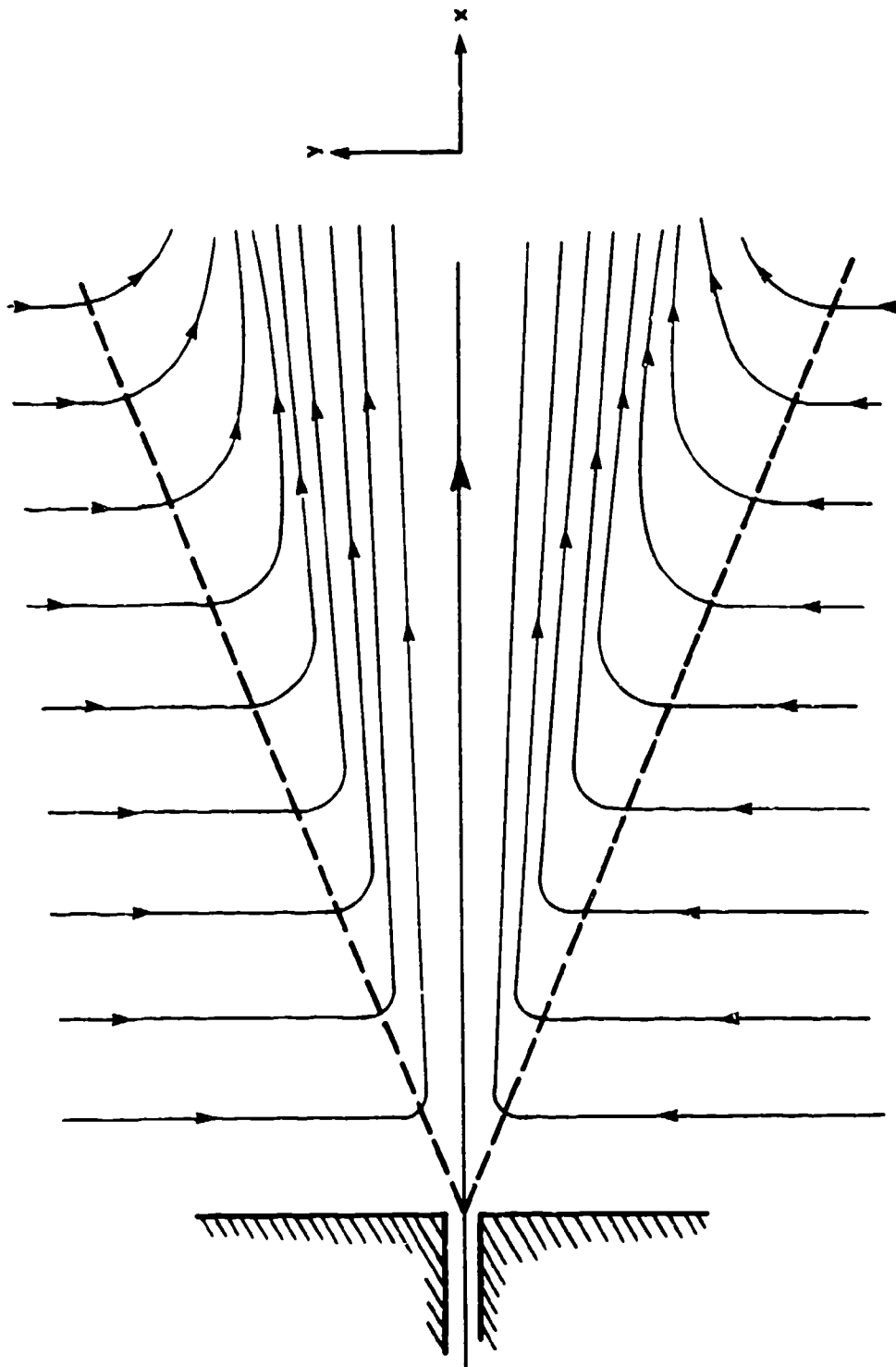


Figure 1. Co-ordinate System for a Circular, Turbulent Free Jet, Illustrating the Pattern of Streamlines.

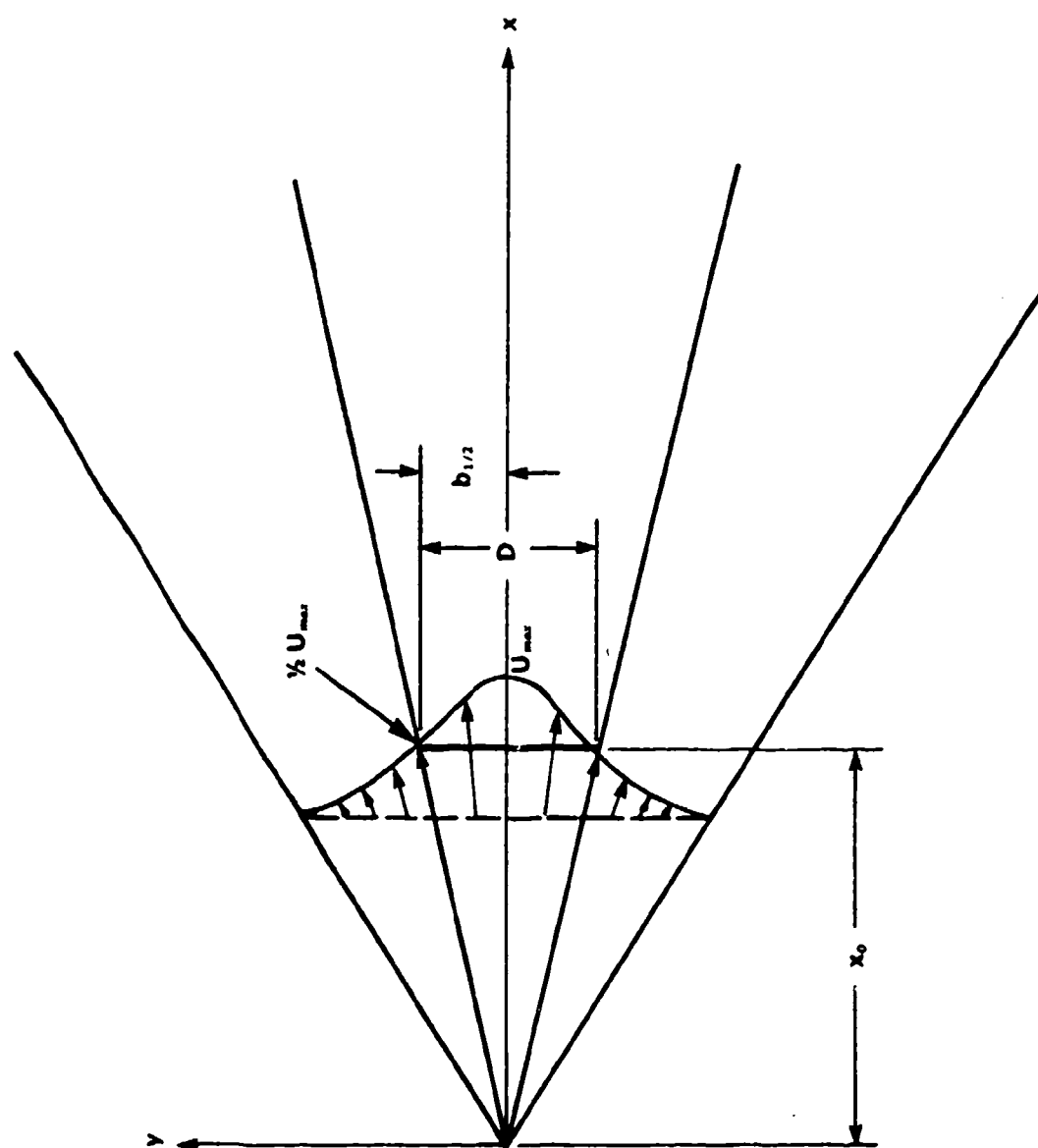


Figure 2. Illustrating the Idea of x_0 with Respect to $b_{1/2}$ and the Jet Diameter D to be Studied.

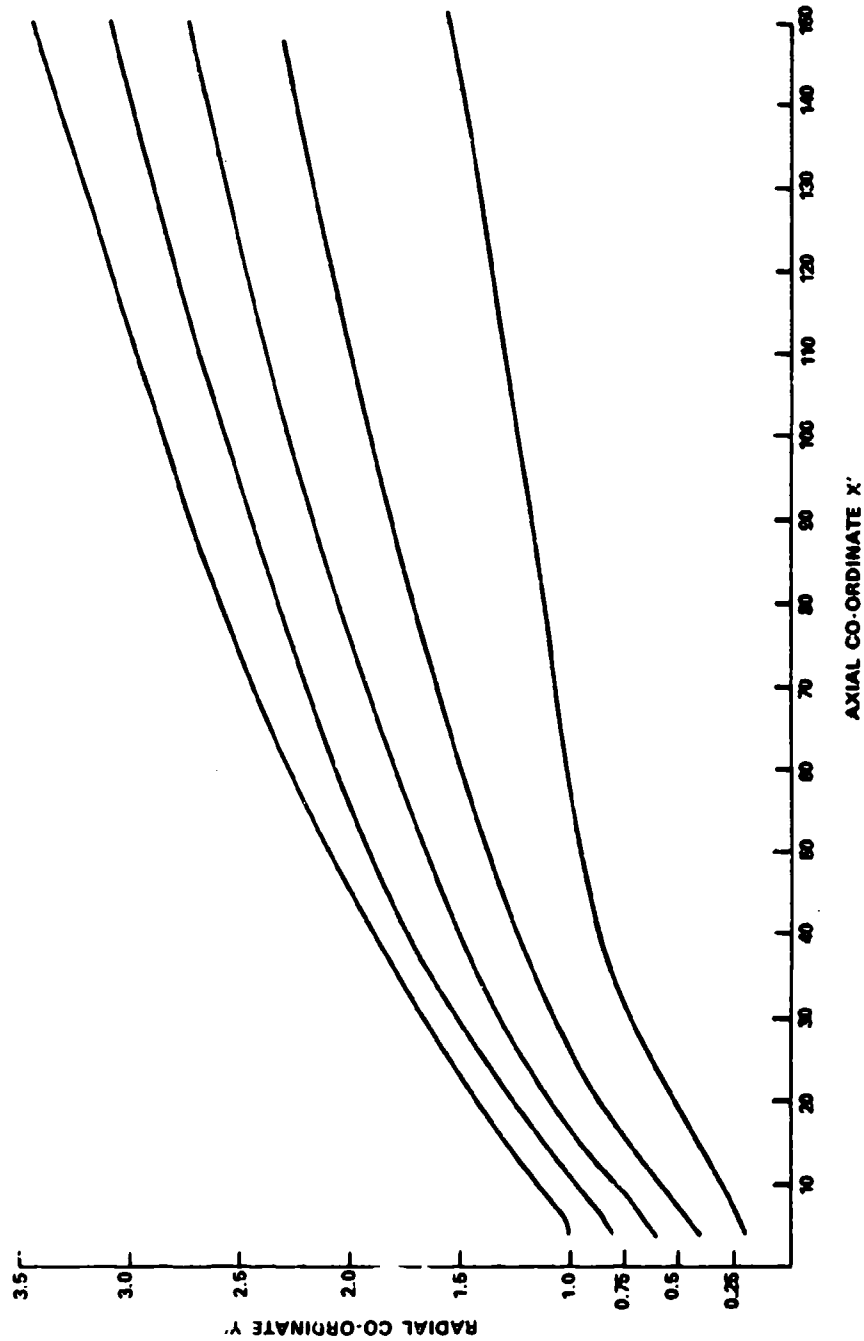


Figure 3. PATHS OF THE PARTICLES IN THE FLUID, FOR A PARTICLE DIAMETER OF 100 μ .

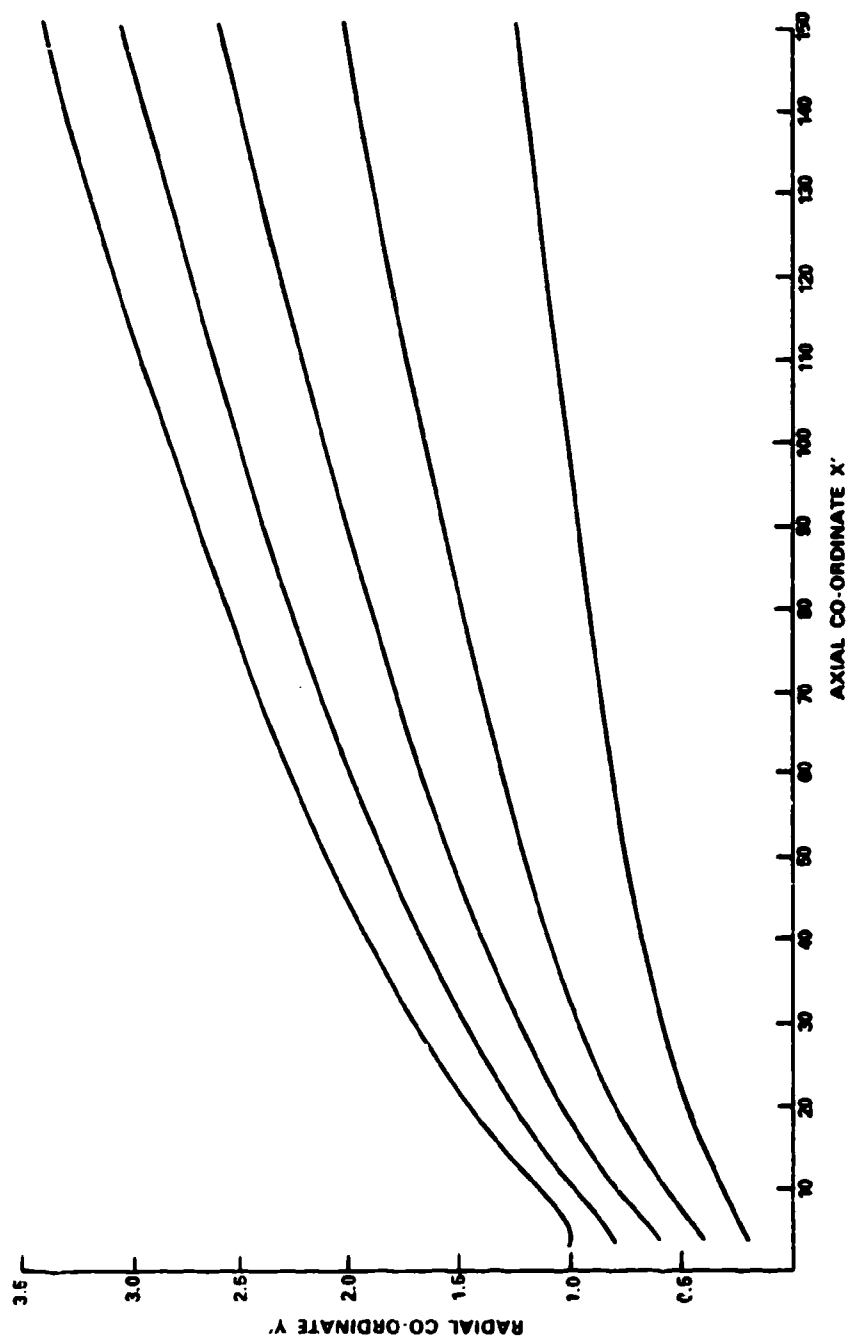


Figure 1. PATHS OF THE PARTICLES IN THE FLUID, FOR A PARTICLE DIAMETER OF 50μ .

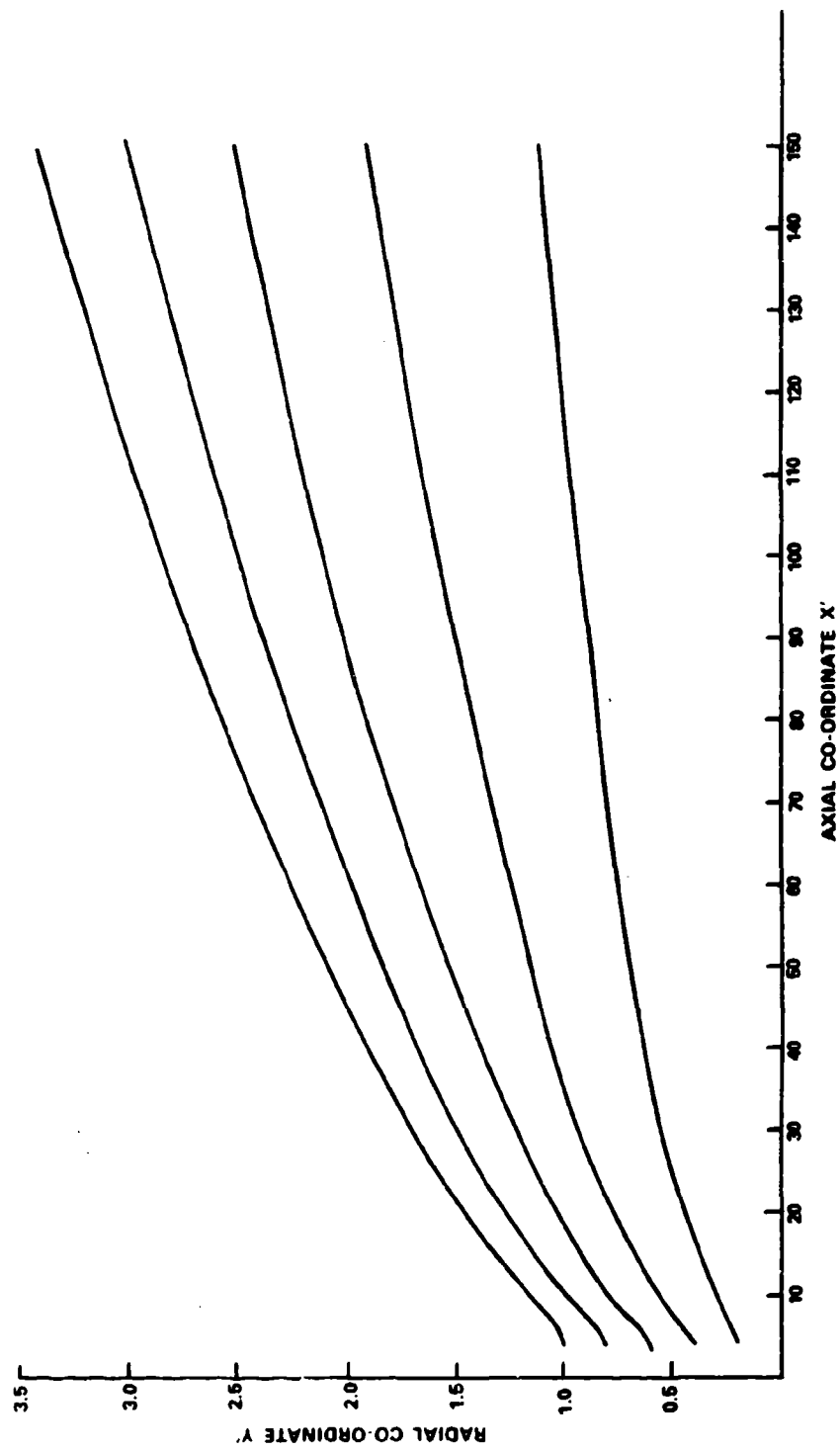


Figure 5. PATHS OF THE PARTICLES IN THE FLUID, FOR A PARTICLE DIAMETER OF $20\ \mu$.

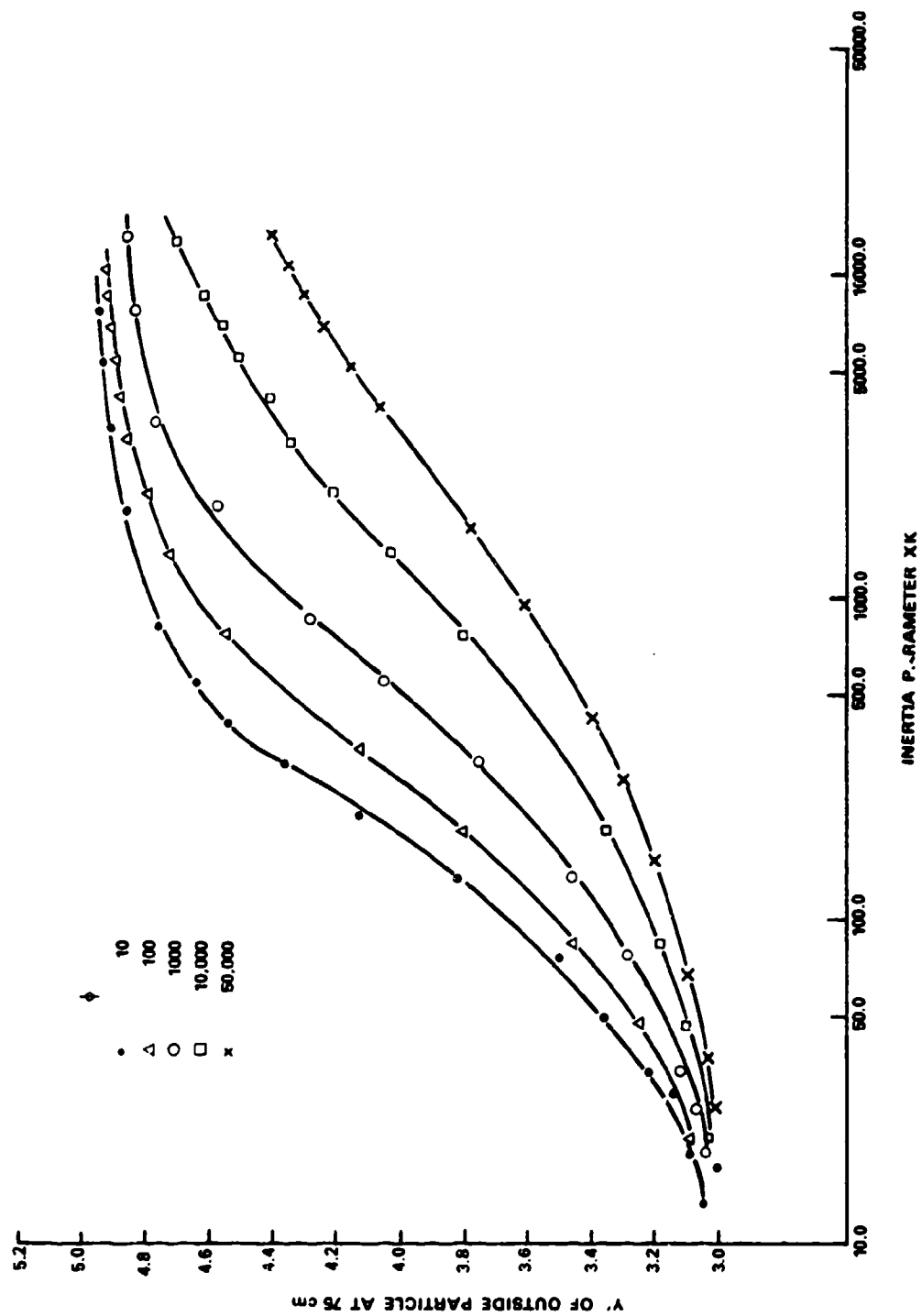


Figure 6. Calculated Radial Co-ordinate of Outside Particle as a Function of its Inertia Parameter, for the Jet Width Resulting when $AF = 1.0$

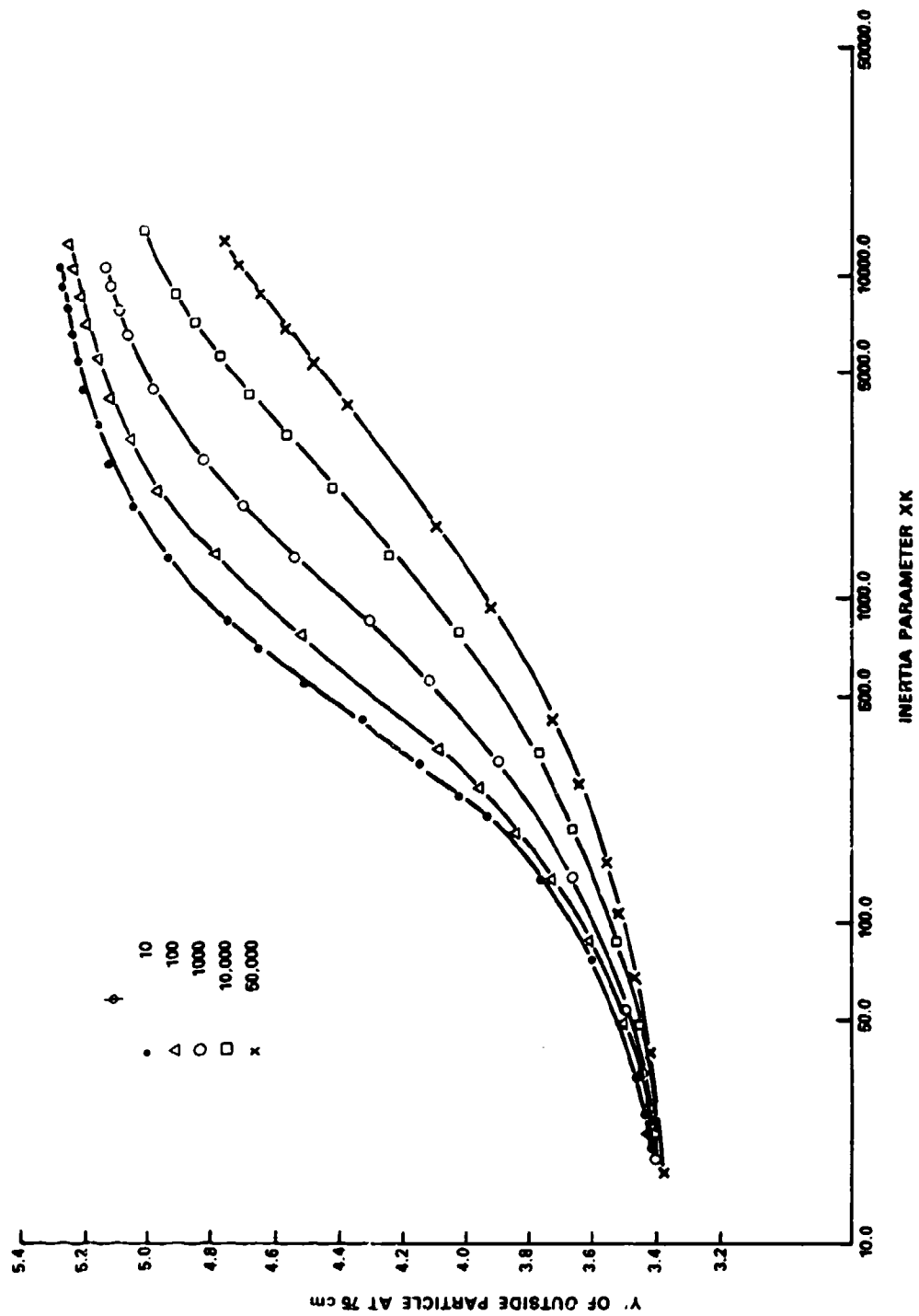


Figure 7. Calculated Radial Co-ordinate of Outside Particle as a Function of its Inertia Parameter, for the Jet Width Resulting when $AF = 0.5$

LEVEL 2.5 (1X06)
 ISN 0024
 MAIN
 US/500 FORTRAN MIXEDUPD
 TO THE INITIAL CONCENTRATION OF PARTICLES IS
 TO THE AVERAGE CONCENTRATION OF PARTICLES. A DISTANCE OF
 PROGRAM IS (.PIU.6)
 .
 ISN 0025
 SUB
 END
 ISN 0026
 EFFECT (PARAMETER) OPTIMIZE LINECOUNT (99) SIZE (MAX) AUTO DBI (NONE)
 OPTIONS IN
 RECALL SOURCE FOCUS MULTI NUCLEI NUCLEAR OBJECT MAP NO FURNAT NUCREF NUALC NUNMSF TERM IBM FLAG (1)
 STATISTICS
 SOURCE STATE-PTS = 25. PROGRAM SIZE = 2214. SUBPROGRAM NAME = MAIN
 NO DIAGNOSTICS GENERATED
 RECALC END OF CALCULATION 000000
 492K BYTES OF CORE NOT USED
 DATE 82-230/12-55-02
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LEVEL 2-3 (FPA06)
REQUIREMENTS: OPTIONS
OPTIONS: IN EFFECT

```

C      SUBROUTINE SBM2(N,F,X,M,IRUNG,M)
C      THIS SUBROUTINE USES THE FOURTH-ORDER RUNGE KUTTA METHOD TO
C      INTEGRATE A FIRST-ORDER ORDINARY DIFFERENTIAL EQUATIONS, ACKUSS
C      ONE STEP OF LENGTH H IN THE INDEPENDENT VARIABLE X. IN THIS
C      CASE, THE U-VALUES TO BE SOLVED ARE THOSE DESCRIBING THE MOTION
C      OF A PARTICLE IN A FLUID.
C      DIMENSION PHI(50),SAVY(50),Y(50),F(50)
C      GO TO (7,3,4,5,6), M
C      PASS 1
C      IRUNG=1
C      RETURN
C      PASS 2
C      DU 22 J=1,N
C      SAVY(J)=F(J)
C      PHI(J)=F(J)
C      Y(J)=SAVY(J)+0.5*H*F(J)
C      X=X+U*H
C      IRUNG=1
C      RETURN
C      PASS 3
C      DU 33 J=1,N
C      PHI(J)=PHI(J)+2.0*F(J)
C      Y(J)=SAVY(J)+U*5*H*F(J)
C      IRUNG=1
C      RETURN
C      PASS 4
C      DU 44 J=1,N
C      PHI(J)=PHI(J)+2.0*F(J)
C      Y(J)=SAVY(J)+H*F(J)
C      X=X+U*5*H
C      IRUNG=1
C      RETURN
C      PASS 5
C      DU 55 J=1,N
C      Y(J)=SAVY(J)+(PHI(J)+F(J))*H/6.0
C      IRUNG=2
C      RETURN
C      END
C      EFFECT=NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(MAX) AU/UBEL(MONE)
C      OPTIONS IN EFFECT=SOURCE CYCLIC NOLIST NODECK OBJECT NOMAP NORFORMAT NOGOSTMT NOTREF NULAL NOANSW
C      STATISTICS= SOURCE STATISTICS= 28. PROGRAM SIZE = 1699. SUBPROGRAM NAME = SBM22
C      DIAGNOSTICS= NO DIAGNOSTICS GENERATED
C      *****END OF COMPILATION*****
C      492K BYTES OF CORE NOT USED

```

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[illegible]

THE DISTRIBUTION OF PARTICLES IN A CIRCULAR JET DOWNSTREAM FROM A JET OPENING

SM 1077

THE PHYSICAL PARAMETERS OF THE FLOW STREAM ARE:

UD = 1.0000
 PU = 0.010000
 KML = 0.011213
 SIGMA = 1.000000
 XMI = 0.0001749
 UF = 700.0000
 MEZ = 47.46225
 XN = 43.4750
 AF = 1.0000

THE NECESSARY DATA FOR THIS RUN IS

ITPL = 11
 NL = 15
 ITP = 51
 M1 = 1
 M2 = 6
 DBP = .20000
 AMAXU = 75.000
 AMAXN = 150.000
 QTAU = 1.0000
 IFOP = 25
 XZ = 11.79200

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THE COMPLETE SOLUTION OF CHOSEN PARTICLES

SM 1077

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THE COORDINATES AND VELOCITY OF ALL PARTICLES AT THE DUMNSINEAM

(ALL VALUES ARE JOURNALIZED: CAUSEU/M,X'Z4/H,V'Y/R,VX'ZVX/UF,VY'ZVY/UF,UX'ZUX/UF,UY'ZUY/UF)

TAU	X'	Y'	VX'	VY'	UX'	UY'	CDME
PARTICLE 1							
THE INITIAL CONDITIONS OF THIS PARTICLE ARE:							
U.U	U.U	U.U	U.U	U.U	U.U	U.U	U.U
25.00	11.792	11.792	0.00433	0.0	0.999439	0.0	0.0
50.00	33.102	33.102	0.058894	0.0	0.956030	0.0	0.0
75.00	49.141	49.141	0.419317	0.0	0.255201	0.0	0.0
100.00	55.114	55.114	0.308299	0.0	0.213837	0.0	0.0
125.00	67.022	67.022	0.249825	0.0	0.190016	0.0	0.0
150.00	72.748	72.748	0.214110	0.0	0.171850	0.0	0.0
175.00	77.428	77.428	0.190605	0.0	0.161824	0.0	0.0
200.00	81.585	81.585	0.174056	0.0	0.152314	0.0	0.0
225.00	85.640	85.640	0.161694	0.0	0.144490	0.0	0.0
250.00	89.176	89.176	0.151924	0.0	0.137472	0.0	0.0
275.00	92.049	92.049	0.143973	0.0	0.132150	0.0	0.0
300.00	95.044	95.044	0.137266	0.0	0.127703	0.0	0.0
325.00	98.273	98.273	0.131566	0.0	0.124113	0.0	0.0
350.00	102.340	102.340	0.126543	0.0	0.121113	0.0	0.0
375.00	105.343	105.343	0.122135	0.0	0.118433	0.0	0.0
400.00	108.292	108.292	0.118173	0.0	0.116430	0.0	0.0
425.00	111.115	111.115	0.114345	0.0	0.115009	0.0	0.0
450.00	113.591	113.591	0.110827	0.0	0.113507	0.0	0.0
475.00	115.935	115.935	0.107622	0.0	0.112131	0.0	0.0
500.00	118.144	118.144	0.104739	0.0	0.110917	0.0	0.0
525.00	121.691	121.691	0.102139	0.0	0.109847	0.0	0.0
550.00	124.619	124.619	0.099893	0.0	0.108907	0.0	0.0
575.00	126.619	126.619	0.097957	0.0	0.108155	0.0	0.0
600.00	129.007	129.007	0.096256	0.0	0.107577	0.0	0.0
625.00	131.144	131.144	0.094756	0.0	0.107126	0.0	0.0
650.00	133.645	133.645	0.093433	0.0	0.106720	0.0	0.0
675.00	135.901	135.901	0.092256	0.0	0.106329	0.0	0.0
700.00	138.117	138.117	0.091194	0.0	0.105944	0.0	0.0
725.00	140.246	140.246	0.090221	0.0	0.105579	0.0	0.0
750.00	142.350	142.350	0.089331	0.0	0.105231	0.0	0.0
775.00	144.550	144.550	0.088524	0.0	0.104894	0.0	0.0
800.00	146.628	146.628	0.087791	0.0	0.104571	0.0	0.0
825.00	148.675	148.675	0.087102	0.0	0.104269	0.0	0.0
THE FINAL CONDITIONS, AT A DISTANCE OF 75.00 DUMNSINEAM, ARE:							
842.00	150.050	150.050	0.086517	0.0	0.103543	0.0	0.0

PARTICLE 2							
THE INITIAL CONDITIONS OF THIS PARTICLE ARE:							
U.U	U.U	U.U	U.U	U.U	U.U	U.U	U.U
25.00	11.792	11.792	0.00433	0.0	0.999439	0.0	0.0
50.00	33.102	33.102	0.058894	0.0	0.956030	0.0	0.0
75.00	49.141	49.141	0.419317	0.0	0.255201	0.0	0.0
100.00	55.114	55.114	0.308299	0.0	0.213837	0.0	0.0
125.00	67.022	67.022	0.249825	0.0	0.190016	0.0	0.0
150.00	72.748	72.748	0.214110	0.0	0.171850	0.0	0.0
175.00	77.428	77.428	0.190605	0.0	0.161824	0.0	0.0
200.00	81.585	81.585	0.174056	0.0	0.152314	0.0	0.0
225.00	85.640	85.640	0.161694	0.0	0.144490	0.0	0.0
250.00	89.176	89.176	0.151924	0.0	0.137472	0.0	0.0
275.00	92.049	92.049	0.143973	0.0	0.132150	0.0	0.0
300.00	95.044	95.044	0.137266	0.0	0.127703	0.0	0.0
325.00	98.273	98.273	0.131566	0.0	0.124113	0.0	0.0
350.00	102.340	102.340	0.126543	0.0	0.121113	0.0	0.0
375.00	105.343	105.343	0.122135	0.0	0.118433	0.0	0.0
400.00	108.292	108.292	0.118173	0.0	0.116430	0.0	0.0
425.00	111.115	111.115	0.114345	0.0	0.115009	0.0	0.0
450.00	113.591	113.591	0.110827	0.0	0.113507	0.0	0.0
475.00	115.935	115.935	0.107622	0.0	0.112131	0.0	0.0
500.00	118.144	118.144	0.104739	0.0	0.110917	0.0	0.0
525.00	121.691	121.691	0.102139	0.0	0.109847	0.0	0.0
550.00	124.619	124.619	0.099893	0.0	0.108907	0.0	0.0
575.00	126.619	126.619	0.097957	0.0	0.108155	0.0	0.0
600.00	129.007	129.007	0.096256	0.0	0.107577	0.0	0.0
625.00	131.144	131.144	0.094756	0.0	0.107126	0.0	0.0
650.00	133.645	133.645	0.093433	0.0	0.106720	0.0	0.0
675.00	135.901	135.901	0.092256	0.0	0.106329	0.0	0.0
700.00	138.117	138.117	0.091194	0.0	0.105944	0.0	0.0
725.00	140.246	140.246	0.090221	0.0	0.105579	0.0	0.0
750.00	142.350	142.350	0.089331	0.0	0.105231	0.0	0.0
775.00	144.550	144.550	0.088524	0.0	0.104894	0.0	0.0
800.00	146.628	146.628	0.087791	0.0	0.104571	0.0	0.0
825.00	148.675	148.675	0.087102	0.0	0.104269	0.0	0.0
THE FINAL CONDITIONS, AT A DISTANCE OF 75.00 DUMNSINEAM, ARE:							
842.00	150.050	150.050	0.086517	0.0	0.103543	0.0	0.0

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<p>A mathematical model has been developed to predict the downstream distribution of particles in a fluid jet. The particles were initially assumed to be evenly dispersed over a cross-section of the circular jet, near the source of the jet, and, with computer-aided methods, the co-ordinates of the particles at a specified distance downstream were determined. The computer program was also built in such a manner that the paths of specific particles, from their initial upstream position to their final position downstream, could be completely followed. Some graphical results of these paths are presented for various particle sizes.</p> <p>(U)</p>		

KEY WORDS

Aerosol
Turbulent Jet
Particulate Dissemination

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